Appendix 1

Mathematics of Diversification

Consider a farmer's decision to allocate effort X among a set of revenue-generating activities such that x_i represents the effort allocated to activity *i*. Each activity *i* is defined by 1) the expected net return per unit effort $E_i = R_i$, where R_i is the per-unit revenue and C_i is the per-unit cost; 2) f_i, a scaling factor on E_i whose variability reflects the effect of disturbance on expected net revenue; as an example, for the cultivation of rice as an activity, a drought might give an $f < 1$, while exceptional growth conditions might give an $f > 1$; and 3) I_i, a fixed investment cost, or transaction cost, for allocating effort in activity *i*. The farmer's profit is given then by:

$$P = \sum_i \left(E_i f_i x_i - I_i \right) \tag{A1.1}$$

By inspection, it is clear that $dP/dx_i \sim E_i$, and thus to maximize profits the farmer allocates effort in activities where E is high and I is not prohibitive. In contrast, $dP/df_i \sim 1/x_i$ and is reduced when variability in f_i is low, or when effort is spread across activities where values of f are not expected to co-vary, keeping x_i in individual activities *i* low.

These objectives of profit maximization and risk minimization may thus be at odds with one another, and it is worthwhile to consider the case of a farmer diverting efforts from an existing set of activities into a new activity *j*. Consider the case where the farmer diversifies effort into a new activity *j*. The change in profit P is given by:

$$\frac{dP}{dx_j} = E_j f_j dx_j - I_j - \sum_{i \neq j} \left(E_i f_i dx_i \right) \tag{A1.2}$$

For simplicity, we assume that allocation away from any activity *i* does not lead to the return of the investment cost I_i. If the change in profit is to be positive, it must be that:

$$
\begin{aligned}
0 &< E_j f_j dx_j - I_j - \sum_{i \neq j} \left(E_i f_i dx_i \right) \\
I_j + \sum_{i \neq j} \left(E_i f_i dx_i \right) &< E_j f_j dx_j \\
\frac{I_j}{dx_j} + \frac{\sum_{i \neq j} \left(E_i f_i dx_i \right)}{dx_j} &< E_j f_j
\end{aligned}
\tag{A1.3}
$$

Noting that effort reallocated to *j*, dx_j, is simply the sum of effort taken away from other activities *i*, we obtain:

$$
\frac{I_j}{dx_j} + Ef_{avg, i \neq j} < E_j f_j \tag{A1.4}
$$

Simply put, if diversifying into *j* is to increase profit, then the expected return on effort (given stressor f_j) must be greater than the average expected return on all other activities (given stressors 1 to *i*) plus the 'per-unit' investment cost to enter into activity *j* given our allocated effort x_j. If we assume that within a set of activities where I_i is not prohibitive to enter, the farmer will invest initially in those activities with higher E_i, it is reasonable to expect that E_j for the new activity *j* will be lower. Thus, $E_j f_j$ will not in general be greater unless f_j is much larger than f_1, f_2…, f_j; in general diversifying effort in order to reduce risk to fluctuations in f_i will lead to a reduction in profit.

The term I_j/dx_j means that, when initial investment costs are higher, the activity requires allocation of more effort in order to be more 'worthwhile,' making diversification a lumpy process, and making many activities with high I_i prohibitive unless E_i is particularly high or variability in f_i particularly low. In cases where the initial investment I_j is high – such as the search for wage labor in a weak labor market, or investment in capital-intensive activities such as cattle or dairy – an unexpected downshift in f_j can leave the farmer significantly worse off than if effort had not been allocated to *j* at all. The factors f_i can reflect many different aspects of climate, markets, and other stressors, and many may co-vary significantly; as such, it is non-trivial for the farmer to estimate whether the lumpy investment of effort into activity *j* is worthwhile or not.

In general, profit maximization involves investing in a smaller number of activities where E_i is high, and I_i is low. On the other hand, risk minimization involves investing in activities where variability in f_i is low, and spreading effort across activities where variabilities in f_i are not expected to co-vary. Given that in general f_i are not perfectly known, and that the investment costs I_i make diversification a lumpy endeavor, it is not trivial for the farmer (with some level of risk aversion) to decide where between these two extremes would be best. Given uncertainties in f_i, it is not obvious whether greater specialization or greater diversification will leave the farmer better off overall.