Appendix 2
Small-world Quotient Calculation

Table A2.1: Calculating the small-world quotient for local, regional, and multi-level networks

Network	Observed average local clustering coefficient	Expected average local clustering coefficient	Observed average path length	Expected average path length	Small-world quotient†
Local	0.227	0.010	8.09	4.93	13.70
Regional	0.173	0.016	3.51	3.64	11.33
Multi-level	0.188	0.009	3.58	3.74	21.97

†Due to rounding of the variables in the table (for display purposes), performing the calculation using the variables above will result in slight variations from the displayed small-world quotients

Table A2.1 illustrates how the small-world quotient is calculated[1], and clarifies why the small-world quotient is largest for the multi-level network. The small-world quotient is obtained by dividing the ratio of observed and expected average local clustering coefficient of a network by the ratio of its observed and expected average path length. The observed values, as the name implies, are the descriptive network statistics obtained from analysis of the empirical network. The expected average local clustering coefficient is calculated by dividing the average degree of a network by the number of nodes in the network, while the expected average path length is calculated by dividing the natural log of the number of nodes by the natural log of the average degree.

Of primary interest here is why the multi-level network possesses the largest small-world quotient, which we hypothesized based on the primary functions of governance, and resulting network structures, we expected would characterize each of the local and regional levels. The relatively large small-world quotient for the multi-level network is partly attributed to the fact the expected clustering is quite low, while the observed clustering remains high; in this scenario, the numerator in the small-world quotient becomes large. At the same time, the expected average path length is low, but not as low as the observed path length; in this scenario, the denominator in the small-world quotient becomes small. The relatively large numerator and small denominator leads to the large small-world quotient in the multi-level network. Stated another way, emphasizing the concepts and not the equation, when cross-level ties are added to the regional and local networks the resulting multi-level network maintains the low average path length observed in the regional network, while the average local clustering coefficient moves toward the high clustering found in the local network. That is, the small-world structure of the multi-level network provides the benefits of both local clustering and regional efficiency.

[1] For a full theoretical and methodological explanation, see: Watts, D. J. 1999. Networks, Dynamics, and the Small-World Phenomenon. *American Journal of Sociology* 105(2):493–527. Another excellent source is Davis, G. F., M. Yoo, and W. E. Baker. 2003. The Small World of the American Corporate Elite, 1982-2001. *Strategic Organization* 1(3):301–326.