**Appendix 1.** Supporting information for research article titled ‘A demographic model to support customary management of a culturally important waterfowl species.’

**Estimation of demographic parameters and associated variances**

*Age-specific numbers of swans*

We estimated the number of hatch-year (HY) and after-hatch-year (AHY) swans at Te Waihora/Lake Ellesmere each year during 2017–2019 based on post-breeding aerial censuses. We conducted censuses on 9 December in 2017 and 2019, and 16 December in 2018. Observers aimed to record birds individually (i.e., in 1s), but if they encountered large groups of swans, they counted in 10s. If swan densities were low, each observer counted HY and AHY swans simultaneously, but if they encountered large concentrations of swans, they counted HY and AHY swans separately during successive flight loops or passes. If counts of either HY or AHY swans within a given pre-determined section of Te Waihora differed between observers by >20% of their combined mean, both observers repeated the census of that section, resulting in two counts of both HY and AHY swans per observer (for more details, see Herse et al. 2020).

The mean of each observer’s total counts of HY swans at Te Waihora differed by <10% of their combined mean for each year. The mean of each observer’s total counts of AHY swans differed by <20% of their combined mean for each year. We used these means of total counts for generating bootstrap distributions of several demographic rates described below.

*Annual per-capita fertility for after-hatch-year swans*

We estimated annual per-capita fertility (number of eggs hatched) for AHY swans ($F_{AHY}$) using information on annual fertility per unharvested nest ($f_0$) and per harvested nest ($f_i$; assumes all first-clutch eggs were removed from harvested nests), and the proportion of AHY swans that attempt breeding ($b$) (see main text, equation 3).

We estimated $f_0$ based on observations of $n = 22$ unharvested nests at the three largest nesting colonies at Te Waihora in 2018 (Herse et al. 2021). This sample of nests excluded nests for which we could not confidently determine fates of any eggs, and nests which had cameras present or different egg removal treatments as part of a separate study (because camera presence reduced rates of egg laying and hatching success; Herse et al. 2021). We used the ‘rpois’ function in R to generate a Poisson-based bootstrap distribution of colony-size-weighted $f_0$, with uncertainty weighted by the colony-specific sample size of nests. Specifically, for each of the three nesting colonies where we collected data, we first generated a bootstrap distribution of $f_0$ as weight*rpois(n = 10000, lambda = fertility*sample), where weight is a weight assigned to the colony based on its size (abundance of nests in 2018, as determined by aerial censuses, described below), $n$ is the number of generated observations (samples), lambda is the estimate of mean fertility or number of eggs hatched per unharvested nest in 2018, and sample is the colony-specific sample size of nests in 2018. We then defined the bootstrap distribution of $f_0$ as the sum of the three colony-size-weighted distributions described previously. Although demographic rates can change through time, the resulting distribution of $f_0$ encompassed estimates from previous studies at Te Waihora, indicating that they were at least realistic (Fig. A1.1).

An experimental customary harvest of swan eggs at Te Waihora in 2018 revealed that during fixed-output harvests, removing entire clutches of eggs had the least impact on population-level hatching success (Herse et al. 2021). Thus, we estimated $f_0$ based on observations of $n = 13$ nests completely harvested as part of the experimental egg harvest (Herse et al. 2021). This sample of nests excluded completely-harvested nests which flooded and/or had cameras present as part of a separate study (because camera presence reduced post-harvest rates of egg laying and hatching success; Herse et al. 2021). These data are from the two largest swan nesting colonies at Te Waihora, which have historically contained a majority
of all nests at Te Waihora (Miers and Williams 1969, Williams 1979, Meis 1988), and been the focal areas for customary egg harvest (Herse et al. 2021). Thus, the model assumes that egg harvest occurs at these colonies. We generated a Poisson-based bootstrap distribution of \( f_b \) using the same approach described above for \( f_0 \).

We estimated \( b \) using information on abundance of nests (\( N_{nest} \)) and AHY swans (\( N_{AHY} \)) at Te Waihora in 2018. We censused nests using a combination of orthophotography of high-density nesting areas, and visual censuses of low-density nesting areas, during the peak-incubation period of the 2018 breeding season (Fig. 1, Fig. A1.2). We used a 50-megapixel Canon digital single-reflex lens (DSLR) camera equipped with a 50-mm focal length lens, mounted on vibration-isolating Sorbothane® mounts and programmed using Avitrix Flight Management System, to photograph high-density nesting areas (Fig. A1.2). Photos were then stitched together into mosaics by SKYVUW Ltd. (Taradale, New Zealand). We counted and marked locations of nests in ArcGIS 10.5 (Environmental Systems Research Institute, Redlands, California) (Fig. A1.2). We estimated \( N_{nest} \) based on the mean of a conservative count of nests for which we were confident in classifying as active at the time of the census (\( n = 840 \)), and a total count that included an additional 31 nests for which we were less confident (\( n = 871 \)). We used the ‘rpois’ function in R to generate Poisson-based bootstrap distributions of \( N_{nest} \) and \( N_{AHY} \). Thus, we defined both bootstrap distributions as \( \text{rpois}(n = 10000, \lambda = \text{abundance}) \), where \( n \) is the number of simulated observations, and \( \lambda \) is the mean count of nests (for \( N_{nest} \)) or AHY swans (for \( N_{AHY} \)). For each back-calculation of estimated survival probability over the first year after hatching (\( S_{AHY} \), described below; Fig. 2) and run of the matrix model (see main text, Fig. 3), we estimated \( b \) as

\[
\frac{2N_{nest}}{N_{AHY}} \quad \text{Eqn A1.1}
\]

Multiplying \( N_{nest} \) by 2 gives the estimated number of nesting swans at Te Waihora because breeding is generally monogamous (Williams 2013).

**Annual survival probability for after-hatch-year swans**

We estimated annual survival probability for AHY swans (\( S_{AHY} \)) using GPS-tracking data collected as part of a separate study (Herse M.R., Lyver P.O’B., Scott N.J., Tylianakis J.M., Ngāi Tahu, unpublished data). We based estimates on known fates of \( n = 17 \) GPS-marked swans captured at Te Waihora in January 2019 (of \( n = 24 \) total swans captured and marked) and monitored for one year. We captured molting swans from a boat using a landing net and fit each bird with a PinPoint GPS Iridium® neck collar we co-designed with Lotek UK (Wareham, United Kingdom) and Lotek NZ (Havelock North, New Zealand). Collars weighed 70 g, which was < 2% of AHY swan body weight (Williams 2013). Of these 17 swans, 14 (82%) survived and 3 (18%) died during the 1-year monitoring period. Causes of mortalities appeared to be unrelated to GPS collars (because swans were alive and active for several months after collars were deployed) or sport hunting (although at least one swan was shot following the 1-year monitoring period). Because annual survival of swans has only two possible outcomes (survived or died), we used the ‘rbinom’ function in R to generate a binomial-based bootstrap distribution of \( S_{AHY} \) based on the parameters from our data. Specifically, we generated the distribution as \( \text{rbinom}(n = 10000, \text{size} = 17, \text{prob} = 14/17) \), where \( n \) is the number of simulated observations (i.e., deployments of GPS-collars), \( \text{size} \) is the number of trials per observation (in this case, the number of collars deployed in 2019), and \( \text{prob} \) is the probability of ‘success’ (i.e., survival probability in 2019). We divided the resulting number of trial successes per observation by the number of trials per
observation to give the probability of success per trial. The resulting bootstrap distribution of $S_{\text{AHY}}$ was centred on estimates from long-term studies of band-recovery data in the 1950s–1980s, and we therefore consider our estimates to be ecologically plausible (Fig. A1.3; Williams 1979, Barker and Buchanan 1993).

Survival probability for hatch-year swans
For back-calculations of $S_{\text{HY}}$ (see main text, equations 4 and 5), we estimated population growth rate ($\lambda$) based on the annual trend in swan population size at Te Waihora during 2017–2019, as determined by aerial censuses. An analysis of a larger census dataset collected at > 100 waterbodies, including Te Waihora, in 2017 found that observer bias was negligible (Herse et al. 2020). Thus, we modelled the population size trend based on the annual mean of the two observers’ counts. Specifically, we estimated $\lambda$ in equation 5 as the exponent of the slope of the linear regression line relating log population size to year (Fig. 3, Fig. A1.4) (Peery et al. 2006). The slope of the linear regression was congruent with that of data collected as part of a community-science annual bird survey led by Waihora Ellesmere Trust (WET; Ford et al. 2017, 2018) (Fig. A1.4). Thus, we assumed that the logarithm of the population growth rate follows a normal distribution and defined a bootstrap distribution of $\lambda$ as $\exp(rnorm(n = 10000, \text{mean} = \text{slope}, \text{sd} = \text{se}))$ where $\exp$ is the exponent function, $n$ is the number of generated observations, $\text{mean}$ is the slope of the linear regression, and $\text{sd}$ (standard deviation) is the standard error of $\text{slope}$ ($\text{se}$).

We applied constraints to back-calculations of $S_{\text{HY}}$ to make them conform with estimated survival rates of swan hatchlings at Te Waihora. In 2018, we fit $n = 50$ hatchling cygnets at Te Waihora with very-high-frequency (VHF) radio transmitters (Model: PipLL ag393, weight: 2.3g, Lotek NZ and Lotek UK) during September–October while cygnets were still in nests. Radio transmitters weighed < 2% of cygnet body weight at the time of tagging. We located marked cygnets approximately weekly using a combination of aerial- and ground-based radio telemetry until VHF transmitter batteries died in December. We coded encounters as ‘seen,’ ‘heard only,’ ‘not detected,’ or ‘dead.’ We used this censore data to develop a Kaplan-Meir (KM) estimator of the probability of surviving from hatching to 40 days ($S_{\text{hatchling}}$; Fig. A1.5), which is the period over which we detected cygnet mortalities prior to all remaining individuals being censoreed. We generated a binomial-based bootstrap distribution of $S_{\text{hatchling}}$ as $\text{rbinom}(n = 10000, \text{size} = 50, \text{prob} = 0.679)/50$ where $n$ is the number of simulated observations (i.e., deployments of VHF transmitters), $\text{size}$ is the number of trials per observation (in this case, the number of VHF transmitters deployed in 2018), and $\text{prob}$ is the probability of ‘success’ (i.e., survival probability in 2018 based on the KM estimator). We divided the resulting number of trial successes per observation by the number of trials per observation to give the probability of success per trial. For each back-calculation of $S_{\text{HY}}$, we sampled from distributions of $S_{\text{HY}}$ and $S_{\text{hatchling}}$ and required that $S_{\text{HY}} < S_{\text{hatchling}}$. We also required that daily survival probability based on $S_{\text{HY}}$ be < daily survival based on $S_{\text{hatchling}}$. To assess the plausibility of the resulting bootstrap distribution of $S_{\text{HY}}$ back-calculations, we used functions available in the ‘survival’ R package (Therneau et al. 2021) to fit survival regressions to the KM curve and extrapolated those regressions to one year after hatching (Fig. A1.5). We fitted regression models using a suite of alternative probability distributions (exponential, Gaussian, logistic, log-logistic, log-normal, and Weibull; Therneau et al. 2021; Fig. A1.5). The bootstrap distribution of $S_{\text{HY}}$ was centred near estimates of cygnet survival probability over the first year based on the most parsimonious survival regression models (Burnham and Anderson 2002), and we therefore consider estimates from the bootstrap distribution to be ecologically plausible (Fig. A1.5).

Modelling of demographic parameters
We estimated $\lambda$ as the dominant eigenvalue of the matrix $A$ (Fig. 3), as described above (Caswell 2001). We used partial derivatives ($\partial$) to calculate sensitivity ($s_x$) of $\lambda$ to an absolute change in each demographic rate $x$ as

$$s_x = \frac{\partial \lambda}{\partial x} = \sum_{i,j} \frac{\partial \lambda}{\partial a_{ij}} \frac{\partial a_{ij}}{\partial x}$$  \hspace{1cm} \text{Eqn A1.2}$$

where $a_{ij}$ is the element of $A$ for row $i$ and column $j$ and $\frac{\partial a_{ij}}{\partial x}$ is the partial derivative of $a_{ij}$ corresponding with $x$ (Caswell 2001). We calculated elasticity ($e_x$) for each demographic rate $x$ as

$$e_x = \frac{x}{\lambda} s_x$$  \hspace{1cm} \text{Eqn A1.3}$$

(Caswell 2001). This approach allowed us to compare sensitivity and elasticity of $S_{HY}$, $S_{AHY}$, and lower-level demographic rates comprising $F_{AHY}$ ($f_0$ and $b$) (Caswell 2001).

To assess sources of uncertainty in $\lambda$, we used variances ($\text{var}$) of bootstrap distributions of $s_x$ to calculate the proportional contribution ($c_x$) of each demographic rate $x$ to the total variance in $\lambda$ as

$$c_x = \frac{s_x^2 \text{var}(s_x)}{\sum_x s_x^2 \text{var}(s_x)}$$  \hspace{1cm} \text{Eqn A1.4}$$

as described by Caswell (2001).

We estimated reproductive value of individual AHY swans (i.e., the lifelong total number of offspring produced, measured as number of fresh eggs laid and number of eggs hatched) following the methods described by Caswell (2001), which incorporate information on age-specific reproduction and life expectancy, and the methods described in the main text (see equation 6). Estimates of life expectancy were skewed due to rare but extremely large values of $\hat{S}_{AHY}$ (close to 1; see Fig. A1.3) being drawn during random sampling. Thus, the estimate of life expectancy for AHY swans that we report in the main text corresponds with the median of the bootstrap distribution of life expectancy.

Literature Cited


Figure A1.1. Density plot and histogram of the initial Poisson-based bootstrap distribution of estimated black swan fertility (number of eggs hatched) per unharvested nest ($f_0$) at Te Waihora/Lake Ellesmere, Aotearoa/New Zealand, in 2018. The vertical dashed line indicates the mean of the bootstrap distribution. Arrows $a$ and $b$ are estimates of $f_0$ in 1960 and 1961, respectively, at Te Waihora (Miers & Williams, 1969), whereas arrow $c$ is an estimate of $f_0$ in 1971 at Te Waihora (Adams, 1971).
Figure A1.2. Images and maps showing how we assessed the total number of nests ($N_{nest}$), and the proportion of after-hatch-year black swans attempting to breed ($b$), at Te Waihora/Lake Ellesmere, Aotearoa/New Zealand, in 2018. Photos A–C show a Canon digital single-reflex lens (DSLR) camera mounted on vibration-isolating Sorbothane® mounts, which we used to photograph nesting colonies through the bottom of a Cessna 185 Skywagon fixed-wing aircraft. Photo D shows an example of the flight transects and image locations for photographing nesting colonies. Photo E shows all the swan nests (red circles) identified at the largest nesting colony, whereas photo F shows a closer view of the nesting colony.
Figure A1.3. Density plot and histogram of the initial binomial-based bootstrap distribution of estimated annual survival probability for after-hatch-year (AHY) black swans at Te Waihora/Lake Ellesmere, Aotearoa/New Zealand, during 2019–2020. The vertical dashed line indicates the mean of the bootstrap distribution. Arrow a indicates an age-distribution-weighted estimate of mean annual survival probability for AHY swans at Te Waihora during 1956–1974 (Williams, 1979), whereas arrow b indicates an estimate of mean annual survival probability for AHY swans at Lake Wairarapa, Aotearoa/New Zealand, during 1974–1989.
Figure A1.4. Raw data (points) and linear regressions (lines) of log black swan population size at Te Waihora/Lake Ellesmere, Aotearoa/New Zealand, related to breeding year. The blue line corresponds with our aerial census data, whereas the red line corresponds with community-science data collected by Waihora Ellesmere Trust (WET; Ford et al., 2017). We assigned each datum to breeding year based on the timing of corresponding nesting season. For example, we assigned both our aerial data collected in December of 2017, and WET data collected in February of 2018, to the year 2017, which is when nesting during the 2017–2018 breeding season occurred. The dashed lines indicate 95% confidence limits.
Figure A1.5. Plot A shows estimates of the probability of surviving to a given age (days since hatching) for juvenile black swans at Te Waihora/Lake Ellesmere, Aotearoa/New Zealand in 2018, which we used to assess whether a bootstrap distribution of estimated survival probability over the first (hatch) year ($S_{HY}$) was ecologically plausible. In plot A, the solid black line indicates a Kaplan-Meir (KM) estimator of survival for $n = 50$ radio-marked hatchlings at Te Waihora in 2018, whereas vertical hatch marks indicate the timing of data censoring, and dashed lines indicate 95% confidence limits. Coloured lines show alternative survival regression models (only the top-ranked models with parsimonious $\Delta$AICc values < 2 are shown). Point a indicates an estimate of cygnet survival probability from hatching to leg-banding age (~3 months) at Te Waihora in 1986 (Meis, 1988). Point b indicates an estimate of cygnet survival probability from hatching to the end of the first band-recovery period (hunting season) at Te Waihora (Meis, 1988; Williams, 1979). Plot B shows the density plot and histogram (flipped along the x axis to aid comparison with plot A) for the back-calculated bootstrap distribution of estimated survival for hatch-year (HY) swans over their first year of life at Te Waihora in 2018. In plot B, the horizontal dashed line indicates the mean of the bootstrap distribution.
Figure A1.6. Plot showing model estimates of number of hatch-year (blue line) and after-hatch-year (red line) black swans, and total population size (i.e., hatch-year plus after-hatch-year swans; black line), at Te Waihora/Lake Ellesmere, Aotearoa/New Zealand, projected across future time (years), beginning in 2018 (i.e., Time = 0). Transient population dynamics die off after about two years, as indicated by the disappearance of oscillations in the trends.